1st Workshop on Relating Logic

Emerging Field: Logic and Philosophy of Science
Department of Logic
Nicolaus Copernicus University in Toruń
Poland

Department of Pedagogy, Psychology, Philosophy
University of Cagliari
Italy

September 25-26 2020

Contents

Introduction

Relating Logic (RL) is a logic of relating connectives (just as Modal Logic is a logic of modal operators). The basic idea behind a relating connectives is that the logical value of a given complex proposition is the result of two things: (i) the logical values of the main components of this complex proposition supplemented with (ii) a valuation of the relation between these components. The latter element is a formal representation of an intensional relation that emerges from the connection of several simpler propositions into one more complex proposition.

More formally, let $A_1, \dots A_n$ be propositions with some fixed logical values and let c be an n-ary relating connective. Then the logical value of complex sentence $\mathsf{c}(A_1, \dots, A_n)$ depends not only on the logical values of $A_1, \dots A_n$, but additionally on the value of the connection between $A_1, \dots A_n$. It therefore depends on an additional valuation of pairs (n-tuples) that is part of the overall process of evaluation of the logical values of complex propositions built with relating connectives. This way we can form logical systems to deal with reasoning about non-logical relationships.

Often when we replace the parameters of classically valid arguments with real sentences and the classical connectives with certain natural language connectives, bizarre inferences result, such as the one below:

$$\frac{\begin{array}{l}\text{Ann has not died or Mark is in despair.}\\ \text{Mark is not in despair or Ann is calling for a doctor.}\end{array}}{\text{Ann has not died or Ann is calling for a doctor.}} \qquad \text{(a)}$$

The problem arises because when we construct everyday arguments, we consider not only the logical values of the sentences but also expect certain non-logical relationships to hold between them, such as a causal relationship in the case above. Further examples of such relationships conveyed by arguments expressed in natural language are analytic, temporal, content, preference and connexive relationships. A formal language needs more than the standard formal apparatus of disjunction and conjunction for handling extensional phenomena; it needs machinery to make sense of intensional phenomena too.

It is easy to observe that if we interpret the expression *or* present in (a) in models $\langle v, R\rangle$ (where v is a binary valuation of variables and R is a binary relation defined on a set of formulas) in the following way: $\langle v, R\rangle \models A \vee B$ iff $\langle v, R\rangle \models A$ or $\langle v, R\rangle \models B$, and $R(A, B)$, then inference (a) is not valid.[1] However, if we assume that R is transitive, then (a) is valid.

[1] Preserving, of course, the classical meaning of negation and writing $\vee$ instead of *or*.

Although the simplest model for a relating logic is a pair: $\langle v, R\rangle$, the situation may get more complicated. We can use multi-relating models to represent more types of non-logical relations between sentences. In addition, the valuation of relationships between sentences may not be binary but may be many-valued or more subtly graded. Furthermore, we can mix relating semantics with possible world semantics, equipping all worlds with additional valuations of complex sentences. Last, but not least, any semantics may be treated as relating one, when we assume that in case of complex sentences a relationship is represented by a universal relation.

The solution that relating logics offers seems to be quite natural, since when two (or more) propositions in natural language are connected by a connective, some sort of emergence occurs. In fact, the key feature of intensionality is that adding a new connective results in the emergence of a new quality, which itself does not belong to the components of a given complex proposition built by means of the same connective. An additional valuation function determines precisely this quality. Talk of emergence is justified here, because the quality that arises as a result of the connections between the constituent propositions is not reducible to the properties of those propositions. Consequently, if the phenomenon of emergence is to be properly captured, we need additional valuations in a model. The key feature of relating semantics is that it enables us to treat non-logical relations between sentences seriously.

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Program

Note that the program uses Central European time.

Day 1 (25.09.20)

11:30 – 12:00 Opening of WRL:

- **prof. Radosław Sojak** (Dean of Faculty of Philosophy and Social Sciences, NCU)
- **prof. Zbigniew Nerczuk** (Director of Institute of Philosophy, NCU)

12:00 – 12:40 **Francesco Paoli**, **Michele Pra Baldi** and **Damian Szmuc**, *Logics of pure variable inclusion*

12:40 – 13:20 **Tomasz Jarmużek**, *Foundations of relating semantics and relating logic*

13:20 – 13:30 Break

13:30 – 14:10 **Mateusz Klonowski**, *History of relating logic. The origin and research directions*

14:10 – 14:50 **Luis Estrada-González**, *Three open issues in relating semantics*

14:50 – 15:50 Lunch break

15:50 – 16:30 **Jacek Malinowski** and **Rafał Palczewski**, *Barbershop paradox and connexive logics*

16:30 – 17:10 **Elisángela Ramírez-Cámara**, *Relating semantics for Nelson's connexive logic NL*

17:10 – 17:20 Break

17:20 – 18:00 **Ricardo Arturo Nicolás Francisco**, *Relating semantics for totally connexive and hyper-connexive logics*

18:00 – 18:40 **Aleksander Parol** and **Jacek Malinowski**, *Boolean connexive logic in the framework of relating semantics*

Day 2 (26.09.20)

12:00 – 12:40 **Alessandro Giordani**, *Basic epistemic relating logic*

12:40 – 13:20 **Piotr Kulicki**, *Towards relating semantics for logic of agency*

13:20 – 13:30 Break

13:30 – 14:10 **Daniela Glavaničová** and **Matteo Pascucci**, *Deontic and causal relationship in definitions of responsibility*

14:10 – 14:50 **Mateusz Klonowski** and **Tomasz Jarmużek**, *Classical monorelational relating logics*

14:50 – 15:50 Lunch break

15:50 – 16:30 **Antonio Ledda**, **Francesco Paoli** and **Michele Pra Baldi**, *Algebraic analysis of demodalised analytic implication*

16:30 – 17:10 **Yaroslav Petrukhin**, *Two three-valued logics inspired by dependence ones*

17:10 – 17:20 Break

17:20 – 18:00 **Víctor Aranda**, *Relating-modal logics and Hausdorff spaces*

18:00 Closing of WRL

Abstracts

Relating-modal logics and Hausdorff spaces

Víctor Aranda
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It is a well known fact that, in the spaces $\mathbb{R}$ and $\mathbb{R}^2$, each one-point set is closed, i.e. each one-point set is its own closure. However, this is not true for arbitrary topological spaces. Some mathematicians believe that topologies in which one-point sets are not closed seem strange (cf. Munkres [4, p. 98]). For this reason, they impose additional conditions that will exclude these *unnatural* spaces. The topological interpretation of the system **S4** — introduced by McKinsey and Tarski ([3]) — does not exclude those models where $\nu(\varphi)$ is a one-point set of the topology and $\nu(\varphi) \neq \nu(\Diamond\varphi)$. Consider, for instance, the following toy model:

- $\langle X, \tau \rangle$, where $X = \{a, b, c\}$; $\tau = \langle \emptyset, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\} \rangle$.
- $\nu \colon P \longrightarrow \mathcal{P}(X)$, where $\nu(\varphi) = \{b\}$.
- $M, w \models \Diamond\varphi$ iff every open set containing w intersects $\{b\}$.

  Let $w = a$. $\{a, b\}$ and $\{a, b, c\}$ are the open sets containing a and $\{a, b\} \cap \{b\} \neq \emptyset$ and $\{a, b, c\} \cap \{b\} \neq \emptyset$, so $a \in \mathsf{Cl}(\{b\})$.

  Let $w = b$. Obviously, $b \in \mathsf{Cl}(\{b\})$.

  Let $w = c$. $\{b, c\}$ and $\{a, b, c\}$ are the open sets containing c and $\{b, c\} \cap \{b\} \neq \emptyset$ and $\{a, b, c\} \cap \{b\} \neq \emptyset$, so $c \in \mathsf{Cl}(\{b\})$.
- Therefore, $\nu(\Diamond\varphi) = \{a, b, c\}$ and hence $\nu(\varphi) \neq \nu(\Diamond\varphi)$.

Aiello et al ([1]) have already pointed to the limitations of the basic modal logic to express many properties of topological interest. In fact, one of the properties they mention is that of being a Hausdorff space. In a Hausdorff space, every one-point set is closed, so a logical system whose models are Hausdorff spaces will exclude every topo-model such that $\nu(\varphi)$ is a one-point set of τ and $\nu(\varphi) \neq \nu(\Diamond\varphi)$. Thus, the question is: What extension of **S4** is needed to deal with Hausdorff spaces?

In this talk, we will suggest a *relating-topo-model* for **S4**. In relating logics (cf. [2]), the standard valuation of variables comes with a *family* of valuation of connections between sentences, one for each connective. Since the modal language includes $\Box$ and $\Diamond$, this family should include a valuation of monadic relations (properties) of sentences. We wanted that, whenever $M, w \models \Diamond\varphi$ holds for $\nu(\varphi)$ a one-point set of τ, $M, w \models \varphi$ also holds (in the model above, this is not true – let $\nu(\varphi)$ be $\{b\}$, and

$w = a$). The property of being a one-point set of τ is a "new quality" which does not depend on the logical values of the constituents of the sentences. Then, it could be imposed as an extra semantic condition:

$$M, w \models \Diamond^{\mathtt{W}^1}\varphi \Leftrightarrow M, w \models \Diamond\varphi \textit{ and } \nu(\varphi) = \textit{one-point element of } \tau. \tag{1}$$

Consequently, $\varphi \in \mathtt{R}^1$ iff $\nu(\varphi)$ is a one-point element of the topology. We can now express the desired result as follows: if $M, w \models \Diamond^{\mathtt{W}^1}\varphi$ holds, then $M, w \models \varphi$ must hold too. The next step is somewhat tricky. Let y be the point of X such that $\nu(\varphi) = \{y\}$. To ensure that $M, w \not\models \Diamond^{\mathtt{W}^1}\varphi$ for every $w \neq y$ requires the existence of an open set containing w not interesecting $\{y\}$. For this reason, it is stipulated that, for each pair w and w' of distinct points of X, there are two sentences φ and ψ which are *connected* iff $\nu(\varphi)$ is a neighborhood of w, $\nu(\psi)$ is a neighborhood of w' and $\nu(\varphi)$ and $\nu(\psi)$ are disjoint. More formally, $\langle\varphi, \psi\rangle \in \mathtt{R}^2$ iff (1) $M, w \models \varphi$ (2) $M, w' \models \psi$ and (3) $\nu(\varphi) \cap \nu(\psi) = \emptyset$. The disjointness of $\nu(\varphi)$ and $\nu(\psi)$ could be also imposed as an extra semantic condition:

$$M, w \models \varphi \to^{\mathtt{W}^2} \neg\psi \Leftrightarrow M, w \models \varphi \to \neg\psi \textit{ and } \nu(\varphi) \cap \nu(\psi) = \emptyset. \tag{2}$$

Notice that in the formula $M, w \models \varphi \to^{\mathtt{W}^2} \neg\psi$ the point w is clearly redundant and can be omitted. The formula $M, w \models \varphi \to \neg\psi$ is saying that the point w does not belong to both $\nu(\varphi)$ and $\nu(\psi)$, but it could be true that $M, w \models \varphi \to \neg\psi$ for some $y \in X$. On the contrary, if the formula $M, w \models \varphi \to^{\mathtt{W}^2} \neg\psi$ is true, then it will be true for every $y \in X$, so $M \models \varphi \to^{\mathtt{W}^2} \neg\psi$. Thus, the relating relation $\mathtt{R}^2$ allows us to express *globality* in the model (what was a difficulty of basic modal languages). In a relating-topo-model (i.e. a topo-model with $\mathtt{R}^1$ and $\mathtt{R}^2$) it can be proved that $\nu(\varphi) = \nu(\Diamond^{\mathtt{W}^1}\varphi)$ or, equivalently, that each one-point set is its own closure. Non-Hausdorff spaces are finally excluded.

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Three open issues in relating semantics

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Although the community of researchers working in relating logic and relating semantics is still small, the field has experienced a fast development in the last decade. In this paper, I make some comments on three topics that can — and perhaps must — be addressed in the near future: a relating treatment of negation, the discussion about the logicality of the relations between formulas, and a relating semantics treating truth and falsity independently both in evaluating connectives and in defining logical validity.

Relating semantics for totally connexive and hyper-connexive logics

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Tomasz Jarmużek and Jacek Malinowski have purported relating semantics for (Boolean) connexive logics [3], [2]. In this paper I investigate the adjustments needed in relating semantics to accommodate hyper-connexivity [5] within Boolean connexive logics. I will offer three options to establish hyper-connexivity and make some observations on the implications for some connexive-related principles spread in the literature of connexivity and put together under the notion of 'totally connexive logics' introduced in [1] and further studied in [4].

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Basic epistemic relating logic

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The aim of this paper is to explore the advantages deriving from the application of relating semantics in epistemic logic. As a first step, I will discuss two versions of relating semantics and how they can be differently exploited for studying modal and epistemic operators. Next, I consider several standard frameworks which are suitable for modeling knowledge and related notions, in both their implicit and their explicit form, and present a simple strategy by virtue of which they can be associated with intuitive systems of relating logic. As a final step, I will focus on justification logic and show how relating semantics helps us to provide an elegant solution to some problems that afflict the standard interpretation of the epistemic explicit operators.

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Deontic and causal relationship in definitions of responsibility

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The present paper will provide an analysis of responsibility in *relating logic*, a novel formal framework systematically developed in [2]. The starting point of our work will be the symbolic definition of various notions involved in the practice of responsibility attribution as proposed in [1]. We will rephrase the mentioned definitions by taking into account deontic relations and causal relations among propositions; this will allow us to obtain a more fine-grained explication of the notions under analysis and, as a consequence, we will be able to argue that the framework of relating logic is very useful for normative reasoning about responsibility.

The formal language employed will consist of a countable set of propositional variables $Var = \{p_1, p_2, p_3, ...\}$, boolean operators, the temporal operators H ("in all possible past states"), and G ("in all possible future states"), an operator for causal contribution $\mathcal{C}$ ("the agent has causally contributed to"), a family of operators of obligation making reference to a normative source O^{s_1}, O^{s_2}, O^{s_3}, etc., and an operator of strict-obligation $\mathcal{O}$; the latter will replace the operator for all-things-considered obligation (O^*) used in [1].

We provide below a sample of the definitions that will be used throughout the presentation (where P is a shorthand for $\neg H \neg$ and F a shorthand for $\neg G \neg$):

Definition 1. ***Prospective Responsibility***
$\mathcal{PR}\phi =_{def} \mathcal{O}\phi$, provided that ϕ does not include any operator for past reference (H or P).

Definition 2. ***Avoidability***
$\mathcal{A}\phi =_{def} PF(\neg\mathcal{C}\phi \wedge H\neg\mathcal{C}\phi \wedge G\neg\mathcal{C}\phi)$

Definition 3. ***Historic Responsibility***
$\mathcal{HR}\phi =_{def} P(\mathcal{C}\phi \wedge \mathcal{A}\phi \wedge \mathcal{PR}\neg\phi)$

In the above definitions, prospective responsibility boils down to a strict obligation; the notion of avoidability is understood in terms of the possibility to never contribute on some ϕ; and historic responsibility is understood in terms of contributing on something avoidable and strictly prohibited. The main novelty of our present work is that both the operator of causal contribution and the operator of strict-obligation will be evaluated according to the semantic clauses for non-boolean operators offered by the framework of relating logic.

The final part of the presentation will be devoted to a discussion of the expressiveness of the two frameworks and to some ideas for further developments of this approach.

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Foundations of relating semantics and relating logic

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This talk is programmatic and introductory. We try to answer the questions of what the relating semantics is and of what relating logic is.

We outline a fine-grained semantics for different kinds of logics. The fundamental idea of the semantics is that the logical value of a given complex proposition is the result of two things: a valuation of propositional variables supplemented with a valuation of relation between the main components of this complex proposition (connection evaluation). The latter thing is a formal representation of intensionality that emerges from the connection of several simpler propositions into one more complex proposition.

In the first part, we present some linguistic motivations for the semantics. Later, we propose a very general, multi-valued view on relating semantics, and, in a more detailed way, we consider its two-valued specification, referring also to its historical applications and origin. A further generalization is made when we combine relating semantics with possible world semantics in the subsequent part.

The paper concludes with a proposal of defining intensional operators as secondary notions that are based on relating connectives. By dint of the proposal, we can control the behavior of the operators by changing properties of semantic structures for the relating connectives that we use in the definitions.

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Classical monorelational relating logics

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A relating model contains two elements: a valuation of propositional variables and a family of connection evaluations (see [1]). Connection evaluations map the Cartesian product of sets of formulas into the set of connection values (e.g. one and zero) and in this way they determine the extent to which (the way in which) the sentences represented by the formulas are related. Clearly, in the case the set of connection values is two-element set (contains only one and zero) the evaluation connection might be identified with a relation.

In the paper we discuss classical monorelational relating logics. Such logics are built by means of the standard classical connectives and the relating counterparts of two-argument classical connectives. They are specified by relating models with the two-element set of logical values and the two-element set of connection values (see [3], [4], cf. [2]). Additionally it is assumed that for all relating connectives there is one corresponding binary relation of formulas. In the paper, we will discuss selected metalogical properties of such logics, including axiomatization and canonical models, as well as indicate some of their possible applications.

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History of relating logic. The origin and research directions

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In the paper the history of relating logic will be presented. First, we will discuss the so-named Epstein's program and the research postulated by it that fits within the framework of relating logic. After that we move on to the description of the Torunian program of relating logic and to identify the main problems and research directions in the field of relating logic.

According to the assumptions of Epstein's program, a logical analysis should take into account two properties of sentences: their logical value and content (see [9, p. 18]). The analysis of the sentential content and the content relationship in terms of Epstein can be presented in a relational approach (by means of the extensions of the so-named relatedness relations) or in a functional one (by means of the so-named union set-assignments) (see [1, pp. 139–143, 156–158], [2, pp. 65–70, 120–123]). Based on a such analysis Epstein defines new non-classical logics: relatedness logics (R and S, see [1], [2, pp. 61–84]) and dependence logics (D, dD, Eq and DPC, see [2, pp. 115–143]). In addition, he determines some non-standard semantics, the so-named set-assignment semantics, for the well-known non-classical logics, e.g. for some modal logics (see [2, pp. 145–287]).

Epstein's program concerns mainly those logics that can be motivated by the analysis of sentential content. Jarmużek and Kaczkowski proposed to extend this perspective by means of relating logics (see [4]). For them the relations between formulas, which are elements of relating semantics, can represent not only content relationships but also various non-content relationships such as causal or temporal ones. Because of that any Epstein's logic might be considered the special case of relating logic.

The good starting point for the analysis of relating logic is the so-named classical monorelation relating logic discussed in [8] (cf. [5]). Logics of this kind are extensions of classical logic received by adding to a propositional language the relating counterparts of conjunction, disjunction, implication and equivalence. A great example of the application of relating semantics is the analysis of Boolean connexive logics, which are a special kind of connexive logics, presented in [6], [7], [10]. Another example can be the definition of propositional operators by means of relating connectives and the introduction of the new kind of philosophical logics as it was suggested in [3] for epistemic operators. The given issues constitute examples of research conducted under the Torunian program of relating logic.

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Towards relating semantics for logic of agency

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The history of logic of agency can be traced back to the 11th century and the works of St. Anselm of Canterbury, who analysed different modes of human actions as the basis of responsibility (see e.g. [15]). In the 20th century an important impact on logic of agency was made by Scandinavian logicians Stig and Helle Kanger [8, 9], Ingmar Pörn [12, 13, 14] and Dag Elgesem [2, 1]. One of their contributions is the theory of *brings it about* operator given in the final form by Elgesem. Its syntactic presentation seems to be clear and natural but it is not easy to find a standard modal semantics for it. An interesting discussion of the problem is given by Guido Governatori and Antonino Rotolo in [4]. Recently a semantical account of the monadic and dyadic *brings it about* operators using neighbourhood semantics was given by Paul McNamara [11].

We present an alternative semantics for agency logic, especially *brings it about* operators. Our approach is based on relating semantics first introduced by Richard L. Epstein [3] and recently developed by Tomasz Jarmużek and others [5, 7, 6, 10]. The semantics makes use of classical valuation for propositional operators in combination with an additional relation R between proposition. A compound proposition is accepted if it is classically true and its arguments are related by R. The specific properties of R define a particular logic.

We reconstruct the monadic and dyadic *brings it about* operators in a language containing a connective of relating conjunction $\wedge^{\mathrm{w}}$ and special action propositions of the form α_i stating that an agent i is in some way active in a situation in concern. Then, the monadic ($BA_i(\phi)$ meaning that an agent i brings it about that ϕ) and dyadic ($BA_i(\psi, \phi)$ meaning that an agent i brings it about that ϕ by bringing about ψ) *brings it about* operators are defined respectively:

$$BA_i(\phi) \equiv \alpha_i \wedge^{\mathrm{w}} \phi$$
$$BA'_i(\psi, \phi) \equiv (\alpha_i \wedge^{\mathrm{w}} \psi) \wedge (\psi \wedge^{\mathrm{w}} \phi)$$

Such a construction of *brings it about* logic gives us immediately a natural and relatively simple semantics. Moreover, it makes it easy to modify the Elgesem's theory and formalise more intuitions of action theory and enrich logic of agency in this way.

Acknowledgements

This research is supported by the National Science Centre, Poland (HARMONIA, *Deontic logic for autonomous vehicles*: UMO-2017/26/ M/HS1/01092)

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Algebraic analysis of demodalised analytic implication

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The logic DAI of demodalised analytic implication has been introduced by J.M. Dunn (and independently investigated by R.D. Epstein) as a variation on a time-honoured logical system by C.I. Lewis' student W.T. Parry. The main tenet underlying this logic is that no implication can be valid unless its consequent is "analytically contained" in its antecedent. DAI has been investigated both proof-theoretically and model-theoretically, but no study so far has focussed on DAI from the viewpoint of abstract algebraic logic. We provide several different algebraic semantics for DAI, showing their equivalence with the known semantics by Dunn and Epstein. We also show that DAI is algebraisable and we identify its equivalent quasivariety semantics. This class turns out to be a linguistic and axiomatic expansion of involutive bisemilattices, a subquasivariety of which forms the algebraic counterpart of Paraconsistent Weak Kleene logic (PWK). This fact sheds further light on the relationship between containment logics and logics of nonsense.

Barbershop paradox and connexive logics

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In 1984 Lewis Carroll published in Mind the paper "A Logical Paradox". http://fair-use.org/mind/1894/07/notes/a-logical-paradox, see also: Storrs McCall, A History of Connexivity, vol. 11 Gabbay – Handbook of the History of Logic. A history of its central concepts 415-451.

Uncle Joe and Uncle Jim are going to a barbershop run by Allen, Brown and Carr, and uncle Jim hopes that Carr will be in to shave him. Uncle Joe says he can prove Carr will be in by an argument having as premisses two hypotheticals. First, if Carr is out, then if Allen is out, Brown must be in (since otherwise there be nobody to mind the shop). Secondly if Allen is out Brown is out (since Allen, after a recent bout of fever, always takes Brown with him). Taking A to stand for Allen is out, B for Brown is out, etc.

Thus we have: (i) If C then (if A then not-B); (ii) If A then B, and these two premises, according to Uncle Joe, imply not-C, because from (i) at least one of them must always be present to mind the shop, and whenever Allen leaves he always takes Brown with him. Now, suppose that Carr is out. In that case then if Allen is out then Brown must be in, in order to tend the shop.

But we know that this isn't true – we've been told that whenever Allen is out then Brown is out. The result is, of course, paradoxical, because under the stated conditions Carr can perfectly well be out when the other two are in, or even when Allen alone is in. The question is, at what point is Uncle Joe's argument fallacious?

Solution is that the two hypotheticals *If A then B* and *If A then not-B* are not incompatible: they may in fact both be true when A is false, as is the case in classical two-valued logic. Hence we cannot infer not-C by modus tollens. The thought underlying this solution is that If *A then not-B* does not properly negate *If A then B*. Burks and Copi disagree, however. When interpreted as causal implications rather than as material implications, the two hypotheticals above are in their opinion incompatible, and this is in general true of causal implication.

In the presentation we apply relating semantics to Lewis Carroll paradox and show its relation to connexive logic based the following Aristotle's and Boethian theses.

(A1) $\sim(A \Rightarrow \sim A)$

(A2) $\sim(\sim A \Rightarrow A)$

(B1) $(A \Rightarrow B) \Rightarrow \sim(A \Rightarrow \sim B)$

(B2) $(A \Rightarrow \sim B) \Rightarrow \sim(A \Rightarrow B)$.

Boolean connexive logic in the framework of relating semantics

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The paper is focused on the results on axiomatization of some class of connexive logics. The presentation of those results will be preceded by introduction of Boolean connexive logics which were created based on the relating semantics (see [1], [2], [3], [5]). Its history and philosophical background will be presented. Then we will unveil the language of Boolean connexive logic and the relating semantics. After laying the foundations, we will introduce the results of axiomatization of some class of Boolean connexive logic and a way of producing canonical models for aforementioned class of logics (see [4]).

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Logics of pure variable inclusion

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Epstein's dependence logic D and dual dependence logic DD [6] are among the earliest and best motivated examples of relating logics (for which see e.g. [8]). These logics aim at formalising a conditional where the content of the consequent is included in the content of the antecedent, or vice versa. In particular, the first degree entailments of Epstein's logics can be so characterised in terms of a variable inclusion constraint:

$$\vdash_{\mathrm{D}} \varphi \to \psi \text{ iff } \vdash_{\mathrm{CL}} \varphi \to \psi \text{ and } Var(\psi) \subseteq Var(\varphi)\,;$$
$$\vdash_{\mathrm{DD}} \varphi \to \psi \text{ iff } \vdash_{\mathrm{CL}} \varphi \to \psi \text{ and } Var(\varphi) \subseteq Var(\psi)\,.$$

These are not the only options for obtaining logics that formalise a notion of content inclusion, though. Such variable inclusion constraints can be attached, rather than to any particular connective, to the consequence relation itself. In [3] and [4], given an arbitrary logic L of type τ, its left variable inclusion companion L^l and its right variable inclusion companion L^r are defined as the logics of type τ whose consequence relations are respectively given by:

$$\begin{array}{ll} \Gamma \vdash_{\mathrm{L}^l} \varphi \text{ iff} & \text{there exists a } \Delta \subseteq \Gamma \text{ such that} \\ & \Delta \vdash_{\mathrm{L}} \varphi \text{ and } Var(\Delta) \subseteq Var(\varphi)\,; \\ \Gamma \vdash_{\mathrm{L}^r} \varphi \text{ iff} & \text{either: } \Gamma \vdash_{\mathrm{L}} \varphi \text{ and } Var(\varphi) \subseteq Var(\Gamma)\,, \\ & \text{or: } \Gamma \text{ contains an L-antitheorem.} \end{array}$$

If L = CL (=classical logic in the type containing $\neg, \wedge, \vee$), then L^l is PWK, Paraconsistent Weak Kleene Logic [7, 5, 2] and L^r is B, Bochvar's Logic [1]. The semantics of both left and right variable inclusion logics are studied in [3] and [4] by a recourse to the technique of *Płonka sums of matrices*.

Observe that L^l contains all the theorems of L, while L^r contains all the antitheorems of L. These preservation results come at a cost: the definitions of L^l and L^r are somewhat inelegant, because they have to provide for such exceptions to the prescribed content inclusion policy. It is therefore natural to consider the *pure left variable inclusion companion* L^{pl} and the *pure right variable inclusion companion*

L^{pr} of an arbitrary logic of type τ, defined as the logics of type τ whose consequence relations are respectively given by:

$$\Gamma \vdash_{L^{pl}} \varphi \text{ iff} \quad \text{there exists a } nonempty\ \Delta \subseteq \Gamma \text{ such that } \Delta \vdash_{L} \varphi \text{ and } Var(\Delta) \subseteq Var(\varphi);$$

$$\Gamma \vdash_{L^{pr}} \varphi \text{ iff} \quad \Gamma \vdash_{L} \varphi \text{ and } Var(\varphi) \subseteq Var(\Gamma).$$

In this talk, we study these logics via the techniques of matrix bundles and Płonka sums of matrices. We also argue that CL^{pl} and CL^{pr} serve better than PWK or B the purpose of joint truth-preservation and meaningfulness-preservation that motivated the work of such logicians as Halldén, Goddard, and Routley.

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Two three-valued logics inspired by dependence ones

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Dependence logics had been defined by Epstein [6] and can be considered as special cases of relating logics, while the idea and program of relating logics have been presented later (see, e.g., Jarmużek's paper [9]). At that one of dependence logics, known as **DAI** (the logic of demodalised analytic implication) was originally introduced by Dunn [5] in the context of relevant logics. Some connection between three- and four-valued logics and dependence ones was established by Demolombe [4]. Later one **DAI** was studied by Baldi [1] in the context of logics of variable inclusion, in particular of those of them which are known as infectious or nonsense logics (a special class of many-valued ones). Also, Baldi presented a matrix semantics based on Płonka sums for **DAI**. The aim of this talk is to further explore the connection between dependence logics and many-valued ones. We introduce two simple logics with the three-valued semantics which might be viewed as certain dependence logics.

Following Ferguson [7], we use the term 'Proscriptive Principle' (PP, for short) which is a requirement for the consequence relation to be such that all propositional variables of the conclusion must be included in the set of all propositional variables of the premises. As follows from Ciuni and Carrara's paper [2], if PP holds and the set of premises Γ classically entails a formula φ, then Γ entails φ in weak Kleene logic $\mathbf{K_3^w}$ [10]. This principle might be viewed as one of possible specifications of Epstein's idea that $\varphi \to \psi$ is true under a valuation v iff not only $v(\varphi) = f$ or $v(\psi) = t$, but also $s(\psi) \subseteq s(\varphi)$, where s is a function from the set of all propositional variables to a set of 'subject-matters'.

On the other hand, one may think about the converse version of PP, i.e. to require that all propositional variables of the premises must be included in the set of all propositional variables of the conclusion. As was shown by Ciuni and Carrara [2], if such a principle holds and Γ classically entails φ, then Γ entails φ in paraconsistent weak Kleene logic **PWK** [8]. Del Cerro and Lugardon [3] suggested a dependence logic with an implication which differs from Epstein's one in the following point: $s(\varphi) \subseteq s(\psi)$ is required instead of $s(\psi) \subseteq s(\varphi)$.

Del Cerro and Lugardon [3] formulated sequent calculi for some of dependence logics. Their rules for conjunction and disjunction are classical, while the ones for the implication and negation are modifications of the classical rules. Taking into account this fact, the connection between Epstein's ideas and PP as well as the connection between dependence logics and many-valued ones, we suggest two three-valued logics having classical conjunction and disjunction as well as non-classical implication and negation satisfying PP (in the first logic) or its converse version (in the second logic). Our logics are combinations of the connectives of strong Kleene logic $\mathbf{K_3}$ [10] (that are the connectives of Priest's logic of paradox **LP** [11] as well) and $\mathbf{K_3^w}$ (that are the connectives of **PWK** as well). Their matrices are as follows:

φ	$\neg$
1	0
$1/2$	$1/2$
0	1

$\vee$	1	$1/2$	0
1	1	1	1
$1/2$	1	$1/2$	$1/2$
0	1	$1/2$	0

$\wedge$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$1/2$	$1/2$	0
0	0	0	0

$\rightarrow$	1	$1/2$	0
1	1	$1/2$	0
$1/2$	$1/2$	$1/2$	$1/2$
0	1	$1/2$	1

The entailment relation is defined via the preservation of the set of designated values. There can be two options for such a set: $\{1\}$ or $\{1, 1/2\}$. The logic corresponding to the first choice we call $\mathbf{K_3^d}$ and the logic corresponding to the second choice we call $\mathbf{LP^d}$. We show that implication and negation of $\mathbf{K_3^d}$ satisfy PP, while the same connectives of $\mathbf{LP^d}$ satisfy its converse. In the talk we plan to present some other properties of these logics, in particular the ones clarifying their connection with relating, classical, and three-valued logics. Additionally, we intend to introduce sound and complete natural deduction systems and cut-free sequent calculi for the logics in question.

Acknowledgements

The research presented in the paper is supported by the grant from the National Science Centre, Poland, grant number DEC-2017/25/B/
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Relating semantics for Nelson's connexive logic NL

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In this paper, I outline a relating logic style semantics for Nelson's logic NL, solving thus an important open problem in connexive logic.

I will offer an overview of both relational semantics for Boolean connexive logics, and the intensional vocabulary included in NL. Then I will go over the process behind obtaining a relational semantics for NL, with an emphasis on the proof for the only contraclassical axiom in the logic. Finally, I will compare the resulting semantics with two connexive logics considered by Jarmużek and Malinowski.

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